Advances in soft matter science enabled by modern techniques in theoretical physics

Five lectures at Shanghai Jiao Tong University, Shanghai, China (July 2013)

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Lecture one: **Introduction to Feynman path integrals**.
Quantum mechanics and the propagation of nonrelativistic particles in one and higher dimensions. Path-integral view of particle propagation. Aharonov-Bohm effect and quantum nonlocality. Classical mechanics as the large-action limit of quantum mechanics: deriving Hamilton's action principle. Imaginary-time path integrals in quantum mechanics. Tunneling: double-well potentials and energy-level splitting, periodic potentials and energy-band formation.

Lecture two: **Directed polymers in two dimensions and quantum many-body physics**.
Systems of mutually repelling directed polymers: issues to consider. Statistical mechanics, partition functions and path integrals. Reinterpretation: paths as imaginary-time world-lines of quantum particles propagating. Polymer repulsion, hard-core bosons and free fermions. Aside: correlations and fluctuations in statistical systems. Fermi gases and polymer density correlations. Quantum amplitudes for partitioning the Fermi sea. Topology, pin constraints and torn fluids. Adding in long-ranged interactions. Exactly solvable quantum many-body problems. Bosonization approach to Tomonaga-Luttinger liquids.

Lecture three: **Directed polymers in three dimensions and quantum many-body physics**.
Partition functions and imaginary-time path integrals revisited. Topology, winding numbers and Chern-Simons theory. Polymer repulsion, hard-core bosons, and fermion/flux-tube composites. Average field approximation and filled Landau levels. Quantum amplitudes for partitioning the Landau sea. Topology, ring constraints and the one-component plasma. Solving the diffusion equation via integration over Brownian paths. Coarse-grained polymers as Brownian paths. Random walks and the central limit theorem. Brownian motion and the statistical mechanics of undirected phantom polymers.

Lecture four: **Statistical mechanics of vulcanized polymers**.
Annealed and quenched randomness. Statistical mechanics and quenched randomness: the replica technique. Repulsive polymers as self-avoiding random walks. The Edwards model of repulsive polymers. Microscopic model of vulcanized polymers and its replica statistical mechanics. From statistical polymer theories to statistical fields theories: field (a.k.a. functional) integrals. Collective coordinates: physical meaning and the reappearance of calculus. Constructing the Landau-Wilson theory.

Lecture five: **Physical properties of emergent equilibrium random solids**.
Symmetries, length-scales and Landau-Wilson theory. Classical (i.e., Landau-style) analysis and its physical implications: stochastic localization. Goldstone excitations: homogeneous and stochastic elasticity. Restoring fluctuations: correlations, percolation, universality and the renormalization group. Coupling to other freedoms: polymer blends and liquid-crystalline polymers. Outlook.